IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Numerical difficulties in obtaining 3D critical exponents from Platonic solids

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1987 J. Phys. A: Math. Gen. 20 5735

(http://iopscience.iop.org/0305-4470/20/16/048)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 01/06/2010 at 16:07

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. 20 (1987) 5735-5736. Printed in the UK

COMMENT

Numerical difficulties in obtaining 3D critical exponents from Platonic solids

F C Alcaraz† and H J Herrmann‡§

† Departamento de Fisica, Universidade Federal de São Carlos, Caixa Postal 676, 13560, São Carlos, São Paulo, Brazil
‡ CBPF, Rua Dr Xavier Sigaud 150, 22290, Rio de Janeiro, Brazil

Received 27 May 1987

Abstract. We test the possibility, originally proposed by Cardy, of extracting critical exponents of 3D systems by exploring the mass gap amplitudes of Platonic solids. We find that this method does not work for the Ising model for numerical reasons.

Conformal transformations (Cardy 1987) between a plane and a strip of finite width provide very useful relations between mass gap amplitudes and critical exponents. Cardy (1985) extended these relations to higher dimensions in special geometries. Specifically, in the three-dimensional system $S^1 \times R$ the correlation length (or inverse mass gap) ξ of an operator of dimension X will scale like $\xi = R/X$ if R, the radius of the sphere, is large. Cardy proposed in his paper the possibility of extracting numerically the critical exponent X by discretising the sphere in a regular way, i.e. approximating it by the Platonic solids: tetrahedron, octahedron, cube, icosahedron and dodecahedron. He also anticipated that because of this discretisation the use of the quantum Hamiltonian instead of the transfer matrix method is more convenient.

We will comment on the use of the quantum Hamiltonian for the case of the Ising model:

$$-\mathscr{H} = \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + \lambda \sum_i \sigma_i^x \tag{1}$$

where σ_i^z and σ_i^x are Pauli matrices placed on the sites of a Platonic solid and the first (second) sum runs over nearest-neighbour links (sites) of the solid. The spectrum of Hamiltonian (1) separates in two disjoint sectors, even and odd, in the basis in which σ^x is diagonal. The ground state is connected to the lowest even (odd) excited state via the dominant thermal (magnetic) operator. Applying Cardy's ideas to a quantum Hamiltonian (Penson and Kolb 1984, Alcaraz and Drugowich de Felício 1984) one finds for large R at the critical coupling λ_c:

$$E_1^e - E_0^e = (d - 1/\nu)(E_1^o - E_0^o) \tag{2}$$

$$E_0^o - E_0^e = \tfrac{1}{2}(d - 2 + \eta)(E_1^o - E_0^o) \tag{3}$$

where $E_n^e(E_n^o)$ is the n-excited energy level in the even (odd) sector.

In order to use (2) and (3) one must know λ_c. Usually it is obtained through the fact that the mass gap $G = \xi^{-1} = E_0^o - E_0^e$ scales at λ_c like R^{-1}. In figure 1 we show the behaviour of G for the first four Platonic solids at various values of λ. One clearly sees that there is no scaling in R^{-1}. Specifically the cube is out of line. We attribute this to the fact that, contrary to, for example, the square lattice at various sizes, the

§ Permanent address: SPh, CEN Saclay, 91191 Gif-Sur-Yvette, France.

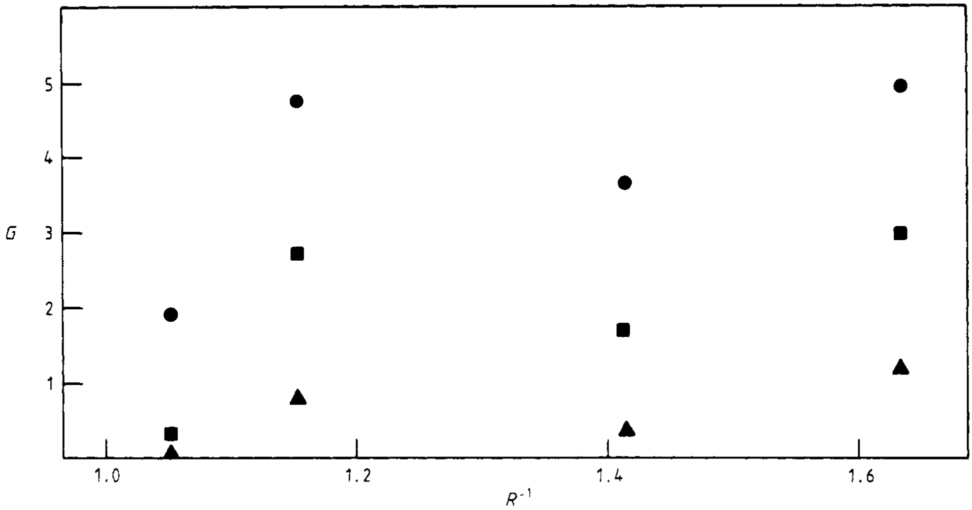

Figure 1. Mass gap G of Platonic solids as a function of R^{-1} where $R=(3/8)^{1/2}$, $1/\sqrt{2}$, $\sqrt{3}/2$ and $[(5+\sqrt{5})/8]^{1/2}$ for the tetrahedron, octahedron, cube and icosahedron for several values of λ (▲, $\lambda=2$; ■, $\lambda=3$; ●, $\lambda=4$).

number of links per site is not the same for the different Platonic solids. This could also be interpreted as the asymptotic λ_c varying from solid to solid. The resulting scattering of the data as seen in figure 1 makes an extrapolation to $R\to\infty$ going from solid to solid impossible.

Although (2) and (3) are only valid for large R, one can see if these equations are approximately consistent for the largest solid that we considered. In this sense one can judge how far one is from the asymptotic regime $R\to\infty$. Using values for the exponents from the literature (Le Guillou and Zinn-Justin 1977, 1980), $\eta\approx0.04$ and $\nu\approx0.63$, one can obtain the values of λ_c for which equations (2) and (3) are consistent with our data for the energy spectrum. For the icosahedron we find $\lambda_c=4.27$, using $\eta=0.04$, from equation (3) and, inserting this in equation (2), $\nu\approx0.68$. On the other hand we get $\lambda_c=4.15$, using $\nu=0.63$, from equation (2) and, inserting this in equation (3), $\eta\approx-0.09$. We conclude from this that the icosahedron is not far from the asymptotic regime $R\to\infty$ for which (2) and (3) should hold. However, we cannot obtain the exponents directly due to the difficulty of extracting the critical coupling λ_c independently.

In summary, we see that the numerical method for extracting exponents of 3D systems proposed by Cardy (1985), which is based on the calculation of the mass gaps of Platonic solids, is numerically unfeasible.

This work was supported in part by Conselho Nacional de Desenvolvimento Científico (CNPq) - Brasil.

References

Alcaraz F C and Drugowich de Felício J R 1984 *J. Phys. A: Math. Gen.* **17** L651

Cardy J L 1985 *J. Phys. A: Math. Gen.* **18** L757

—— 1987 *Phase Transitions and Critical Phenomena* vol 11, ed C Domb and J L Lebowitz (New York: Academic) p 55

Le Guillou J C and Zinn-Justin J 1977 *Phys. Rev. Lett.* **39** 95

—— 1980 *Phys. Rev.* B **21** 3976

Penson K and Kolb M 1984 *Phys. Rev.* B **29** 2834